CONTRADICTORIES AND ENTAILMENT

(1)

It is well known that Russell analyzed the sentence 'The King of France is wise' roughly as follows: 'there is one and only one person who is king of France and he is wise.' In other words according to Russell's analysis the proposition

(A) 'the king of France is wise'

entails the proposition

(B) 'there is a king of France.'

Similarly the proposition

(A′) 'the king of France is not wise'

entails

(B) 'there is a king of France.'

Now if (A) entails (B) the contradictory of (B) must entail the contradictory of (A).

Hence

(B′) 'there is not a king of France'

entails

(X) 'it is not the case that the king of France is wise.'

Suppose now we were to identify (X) with the proposition (A′), i.e. 'the king of France is not wise,' as ordinary usage might tempt us to do.

Then we should have to say that

(B′) 'there is not a king of France'

entails

(A′) 'the king of France is not wise'

which entails

(B) 'there is a king of France.'

But (B′) and (B) are incompatible.

It is well known, of course, that Russell avoids this situation, because on his analysis (X) and (A′) can *not* be identified, and (A) and (A′) are not contradictories.

Suppose now that we do not analyze 'the king of France is wise' in Russell's way, but adopt what might be called the Strawsonian approach.[1]

[1] Cf. P. F. Strawson, 'On Referring,' *Mind*, Vol. 59 (1950) pp. 320–344. Also P. T. Geach 'Subject and Predicate.' *Mind*, Vol. 59 (1950) pp. 461–482, P. T. Geach 'Russell's Theory of Descriptions.' *Analysis*, Vol. 10, (1949–50) pp. 84–92.

According to this we do not regard the statements 'the king of France is wise' and 'the king of France is not wise' as proper statements unless there is a king of France. In other words we do not regard 'Is the king of France wise? Yes or No' as a proper question. (According to Russell's analysis it is a proper question, the answer to which is 'No.')

On the Strawsonian analysis (A) and (A′) now do become contradictories. It now follows that we must give up the assertion that (A) entails (B) (or (A′) entails (B)). For if we adopted the rule that (A) and (A′) are contradictories and did not give up the rule that 'the king of France is . . . .' entails 'there is a king of France' we should have the following situation: —

(A) 'The king of France is wise'
entails
(B) 'there is a king of France'
So
(B′) 'there is not a king of France'
entails
(A′) 'the king of France is not wise'
which entails
(B) 'there is a king of France.'

In other words (B′) would entail its contradictory (B), which is absurd.

Hence if we take the Strawsonian approach and regard (A) and (A′) as contradictories we must give up the rule that (A) entails (B). And this is, of course, exactly Strawson's position, though he does not give exactly this argument for it, but rests his case largely on an appeal to ordinary usage.

While (A) does not entail (B) there is certainly a sense in which a person who says (A) *implies* (B), just as a person who says that the house is on fire implies that he thinks the house is on fire, though the statement that the house is on fire does not entail that so-and-so thinks that it is.[2]

So far we have been covering well-known ground. But there are analogous cases which are a little surprising.

(2)

Many mathematicians would be inclined to say that '$x < 12$' and '$x \geqslant 12$' are contradictories. They would also be inclined to say that

'$x < 12$' and '$x \geqslant 12$'
each entail 'x is a number.'
But clearly these two views are incompatible.
For if
'$x < 12$' entails 'x is a number'

[2] On this use of 'implies' see G. E. Moore in *The Philosophy of G. E. Moore*, edited by P. A. Schilpp, pp. 540–543.

then

'x is not a number'

entails

'x is not < 12'

i.e.

'$x \geqslant 12$'

which entails

'x is a number.' So 'x is not a number' entails 'x is a number.' Which is absurd.

So we must give up saying that '$x < 12$' and '$x \geqslant 12$' are contradictories or we must give up saying that '$x < 12$' and '$x \geqslant 12$' entail 'x is a number.'

The former course would lead to great difficulties and inconveniences. In fact in a good notation, for example in current systems of mathematical logic, the latter course is taken. Indeed in these systems the expression 'x is a number' is not even allowed. But by suitable conventions the variable in '$x < 12$' *shows* that it is a number-variable.

If we allow 'x is a number' as a significant expression, it occurs as an expression of the informal exposition of proofs. It cannot occur within the actual proofs, and can not stand in entailment relations with the actual mathematical expressions. But someone who says '$x < 12$' of course *implies* (in Moore's sense) that x is a number.

(3)

Does 'this is a caricature' entail 'this is a drawing'? It might be held that it does. It might also be held that the sentences 'this is a caricature' and 'this is not a caricature' can be properly used to make a statement only if 'this' refers to a drawing of some sort. Yet these two positions are incompatible. For suppose we accept both. Then 'this is a caricature' and 'this is a drawing other than a caricature' become contradictories. Also if

'this is a caricature' entails 'this is a drawing'

then

'this is not a drawing' entails 'this is not a caricature.' But 'this is not a caricature' entails 'this is a drawing.' So 'this is not a drawing' entails 'this is a drawing.' Which is absurd.

Here the decision whether to give up treating 'this is a caricature' and 'this is a drawing other than a caricature' as contradictories or to give up the entailment of 'this is a drawing' by 'this is a caricature' is a fairly open one. It is the same decision as whether we are to regard *drawing* as a genus or as a category. It is the same decision as that of whether we are to treat 'this is not a caricature—it is a hippopotamus' as significant. (The decision as to whether to regard a concept as a genus or a category is of course more or less arbitary unless we are dealing with a formalized system.)

(4)

Clearly there is an indefinite number of cases of the same sort.

Does 'x is red' entail 'x has a color'?

Does 'x is square' entail 'x has a shape'?

Does 'x ought to do y' entail 'x can do y'?

These are left as exercises to the reader. There are clearly special difficulties about the last one. For a person could clearly hold that 'x ought to do y' and 'x ought not to do y' are properly used to make statements only where x *can* do y, and still deny that 'x ought to do y' and 'x ought not to do y' are contradictories. Indeed there is a special reason why one should hold the latter position. For it is plausible to hold that 'x ought to do y' entails that 'x has a duty' and that 'x ought not to do y' entails 'x has a duty' (to refrain from y). So if 'x ought to do y' and 'x ought not to do y' were contradictories 'x has not a duty' would entail 'x ought to do y' which entails 'x has a duty.'

(5)

To sum up: there are many cases where we have the following alternatives.

(a) 'x is a ϕ' and 'x is a not-ϕ' make sense if and only if x is a ψ, and 'x is a ϕ' and 'x is a not-ϕ' are proper contradictories. Then 'x is a ϕ' (or 'x is a not-ϕ') must not be said to entail 'x is a ψ.' ψ is a category under which the concept ϕ falls. It is not a genus of which ϕ and not-ϕ are species.

(b) ψ is a genus under which φ and non-φ are species. Then 'x is a φ' and 'x is a non-φ' are not contradictories, and they each entail 'x is a ψ.'

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